Could it be?: Children's Representation of Multiple Possibilities

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The ability to reason about multiple possibilities is central to human cognition. We reason about possibilities when making a choice between two options, weighing the pros and cons to each alternative (Tversky & Kahneman, 1988), when engaging in hypothesis testing (Lapidow & Walker, 2020), or when generating multiple plausible causes for a particular event (Gopnik & Wellman, 2012; Meltzoff et al., 2012). We also represent multiple possibilities when thinking counterfactually about what might have happened in the past (Johnson-Laird et al., 1992; Kahneman & Tversky, 1981) or when reasoning about what might happen in the future (Beck et al., 2006).

According to many theories of cognitive development (Gopnik & Wellman, 1994; Xu & Griffiths, 2011; Xu, 2019; Gopnik & Tenenbaum, 2007; Perfors et al., 2011), the capacity to reason about multiple possibilities is crucial for early learning. Based on these accounts, young learners must be able to implicitly generate a potentially infinite number of possibilities, weighing the relative likelihood of each, in order to explain the complex world around them (Gopnik & Wellman, 2012). Research shows that even infants can engage in this ability through statistical learning (Saffran et al., 1996; Xu et al., 2013; Teglás et al., 2011), and that young children can generate and discriminate hypotheses through processes that approximate Bayesian reasoning (Gopnik et al., 2001; Gopnik et al., 2004; Perfors et al., 2011). However, there is contrasting evidence regarding whether young children are capable of employing possibility concepts. While one line of work claims this ability emerges later in development due to its dependence on the acquisition of modal concepts (e.g., "possible," "necessary") (Leahy & Carey, 2020), infant research provides some initial evidence that this ability might be early emerging (Gopnik et al., 2001; Gopnik et al., 2004; Denison & Xu, 2013; Gweon & Schulz, 2011).

Several studies suggest that the capacity to reason about multiple possibilities is early to emerge in infancy. In a violation-of-expectation paradigm, four- and six-month-old infants watched as an experimenter pulled a sample of ping pong balls out of boxes that contained mostly yellow, and few pink balls (Denison & Xu, 2013). Six-month-olds looked significantly longer and were more surprised when they saw an improbable outcome (e.g., four pink balls and one yellow ball) compared to a probable one (e.g., four yellow balls and one pink ball). This suggests that infants may be able to compute probabilities over different alternatives from a young age (see also Denison & Xu, 2010; 2014; 2019).

In another study, 16-month-olds were able to determine the correct causal inference to explain why a toy failed to work based on different cues they observed. An experimenter first showed participants a green toy that always played music with the press of its button. Children witnessed this event three times before being handed either the same green toy or a yellow toy that shared the same shape. The experimenter also placed a similar red toy close to the infant. Critically, neither the green toy nor the yellow toy played music when the child pressed the button. In order to reason about this problem, children might generate multiple potential hypotheses to explain the inconsistency. They found that children who were handed the green toy were significantly more likely to infer that they lacked the ability to operate the toy and enlisted the help of another agent such as a parent or the experimenter. Contrastingly, children who were given the yellow toy were more likely to reach for the red toy, suggesting their hypothesis for the yellow toy's lack of music was due to the toy itself, therefore demonstrating an early sensitivity to the likelihood of one possibility over another (Gweon & Schulz, 2011).

Other lines of work have found that infants look longer and have greater pupil dilation during violation-of-expectation paradigms that employ concepts of disjunctive syllogism (i.e., A or B, not A, therefore B) (Cesana-Arlotti et al., 2018; Cesana-Arlotti et al., 2022). Infants also look longer when presented with an improbable physical outcome compared to a probable one (Teglás & Bonatti, 2016). This evidence suggests that the ability to distinguish plausible outcomes and their respective probabilities from implausible outcomes is present from a young age.

In addition to infant successes, a growing body of work suggests that preschoolers may be capable of employing possibility concepts. In one particular study, children were asked to retrieve a specific colored gumball from one of two transparent gumball machines. One machine contained one gumball, while another machine contained two differently colored gumballs, one of which was the same color as the gumball inside the singleton machine. When prompted to retrieve a particular color (that matched the color inside of the singleton machine), children performed almost entirely at ceiling and correctly selected the gumball machine with only one gumball inside, thus suggesting some sensitivity to both probability and possibility concepts (Alderete & Xu, 2023).

Despite this early competency in infancy, preschool-aged children tend to fail most other tasks which require them to explicitly represent alternative possibilities. In Mody and Carey's

(2016) Cups Task, an occluder covers a singleton cup and a pair of two cups while an experimenter places a sticker into the singleton cup and then into one of the cups from the pair. The child is asked to choose one of the cups in order to win the sticker reward inside. Despite the fact that the singleton cup is 100% guaranteed to have a sticker, three-year-olds fail to maximize their rewards in this task, picking the singleton cup only 60% of the time (Mody & Carey, 2016). A similar effect persists even when children are given the opportunity to discard an option (Leahy et al., 2022) and when the working memory demands of the task are reduced by using see-through tubes instead of cups (Leahy, 2023). Children also seemingly fail to apply possibility concepts when attempting to catch a marble as it falls down a set of opaque y-shaped branching tubes; children younger than four years failed to cover both exits and instead only used one hand to cover one of two tube openings (Redshaw & Suddendorf, 2016).

A number of reasons might explain why preschoolers often fail tasks that require them to reason about alternative possibilities. One explanation suggests that because modal concepts (i.e., "possible," "necessary") are not acquired until around the fifth year of age, younger children are only minimal representers of possibility (Leahy & Carey, 2020). According to the minimal account, children are unable to mark something as merely possible, and when faced with two possibilities, will build a simulation about which one might be correct and will treat that simulation as true knowledge until otherwise proven incorrect. This account would explain why children only choose the singleton cup about 60% of the time (Leahy & Carey, 2019) and why they only use one hand to catch a marble (Redshaw & Suddendorf, 2016).

Despite this compelling account from the literature, it is possible that previous task designs masked preschoolers' true ability to reason about possibilities. In Redshaw and Suddendorf's task, for example, the demands of the task may have been unclear (i.e., whether children should guess where the marble would fall or whether they should catch the marble) (see Turan-Küçük & Kibbe, 2024 for another alternative explanation). Additionally, children's failures on the cups task (i.e., picking a merely possible cup instead of a guaranteed one) may have been a result of their strong motivation to explore. Evidence suggests children have a high motivation to explore (Liquin & Gopnik, 2022; Schulz & Bonawitz, 2007; Gopnik et al., 1999), and information gain might be just as rewarding as obtaining a tangible reward (Liquin & Lombrozo, 2020; Lapidow et al. 2022).

Because children's motivation to explore might account for previous results on the Cups Task and related designs, the current set of studies remove this potential confound by eliminating the need for children to make a forced choice between a necessary and a possible outcome. We designed three experiments with a novel search paradigm. In Experiment 1, three- and four-year-olds (N = 54) watched as a target object fell down a set of either fully transparent or fully opaque sets of y-shaped tubes into rectangular boxes during two separate, within-subject trials. The transparent set of tubes allowed children visual access to the target object as it fell, yielding only one necessary location, whereas the opaque tubes blocked visual access, yielding two equally possible locations. Critically, a piece of fabric prevented the target object from actually falling into one of the boxes. Children were then instructed to search for the target object under a 31 second time constraint. If children's abilities were masked in previous designs, and children are sensitive to the presence of multiple alternative possibilities, they should spend less time searching in the first location they approach when the object is dropped from the fully opaque tube, compared to when it is dropped from the fully transparent one. Children should also switch boxes sooner in the opaque trial compared to the transparent trial. The results from Experiment 1 confirmed these hypotheses. 3-year-olds searched significantly longer in the first location they approached in the transparent (M = 24.02, SD = 9.77) compared to the opaque trial (M = 14.02, SD = 11.21), $t(27) = 4.468$, $p = .0001$. The same was true for 4-year-olds, (transparent: M = 20.54, SD = 11.366; opaque: M = 7.10 , SD = 6.1374), $t(24) = 5.9564$, $p < .001$. These results suggest that three- and four-year-olds are sensitive to the presence of multiple, mutually exclusive possibilities.

Experiment 2

In order to further investigate the findings from Experiment 1, we designed an iPad task that replicates Experiment 1's findings in a digital format. It is possible that children's search behavior in Experiment 1 was guided by sensory feedback from physically searching inside boxes. The digital task provides less sensory feedback, and allows for further iterations of the study (see Experiment 3).

Experiment 2 Methods

Participants

48 preschool aged children (N = 24 three-year-olds, N = 24 four-year-olds) were recruited from the Early Learning and Cognition Lab's existing database, museums, and preschools in the greater San Diego area. Participants were both monolingual English speakers and bilinguals whose primary language is English. 6 additional children were tested but excluded from data analysis due to withdrawal from the study (N = 3), and technical errors with the iPad during test trials (N = 3). Families who participated in the lab received a $5 gift card as compensation for their time.

Materials

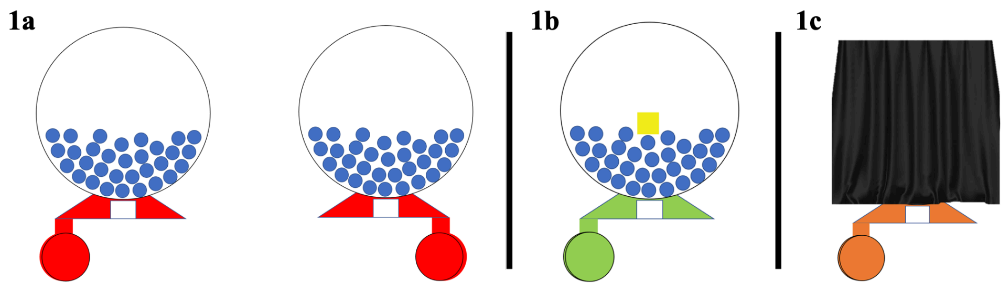
Materials included a 12.9 inch display iPad Pro with a kickstand case, a Bluetooth remote control clicker, a child-size mitten, a laminated dark circle, 1.5 inch smiley face stickers, and animal-shaped 3D stickers. Stimuli were designed using PowerPoint.

Procedure

Children sat to the left of the experimenter and had a glove placed onto their non-dominant hand. Their non-dominant, gloved hand was placed off to the side of the table, while their dominant hand was placed onto the laminated dark circle on the table centered in front of the iPad. Children were instructed to only use their dominant hand throughout the study in order to ensure children's decision to switch boxes was purely intentional (having access to two hands limits this intentionality). Additional prompts were provided as needed for this rule.

Warm-up Phase: The experimenter introduced the target object (a yellow square) to the child, explaining that their goal was to find as many of the yellow squares as they can. The more yellow squares they found, the more stickers they would win. The screen then showed the child two gumball-like machines filled with blue marbles (See Figure 1a). Children observed that when a button connected to each machine was pressed, one of the blue marbles would be released from that machine at the bottom. Children had a chance to press each machine's button once. A new machine then appeared and children watched as a yellow square dropped into the machine (See Figure 1b). Children were then asked to press the button to make the yellow square come out. In this trial, three blue marbles were dispensed from the machine before the yellow square. Then, a new machine appeared, though this time, an occluder prevented the child from seeing the contents inside of the machine (See Figure 1c). A yellow square was dropped into this machine, and children pressed the button to make the yellow square come out. In this trial, four blue marbles preceded the yellow square.

Figures 1a, 1b, and 1c



Machines presented during Warm-up Phase

Familiarization 1: Two new occluded machines appeared on the screen (See Figure 2). This time, the child was told that a yellow square had already been hidden inside one of the machines. Children were instructed that they can only use their dominant hand, but could search inside of both machines. The yellow square was hidden in the left machine and was dispensed after six presses. If the child expressed difficulty or had pressed the same machine's button 15 times, they were reminded that there is only one yellow square but two machines. Children were given a second prompt to remind them that they may search "one machine, the other machine, or both machines." If the child continued to experience difficulty or states they cannot find it in one of the machines, the experimenter provided a third prompt asking the child where else the yellow square could be. When the child succeeded, they received one of the 1.5 inch smiley face stickers.

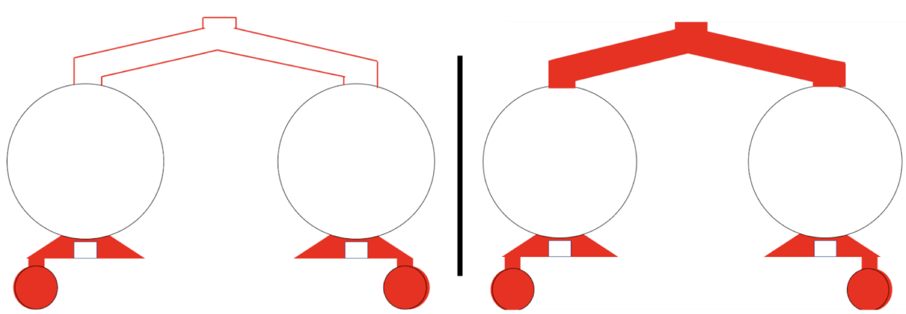
Familiarization 2: Familiarization 2 followed the same procedure as Familiarization 1, but the yellow square was hidden inside of the right machine and appeared after two presses. This trial also introduced a 31 second musical timer. In order to win a sticker, the child had to find the yellow square before the timer ends. If the child found the sticker before the timer ended, the experimenter waited until the end of the music before moving on in order to ensure the child had experience with the length of the timer. The experimenter also referred to the timer when giving feedback at the end of this trial – either by saying "Good job! You found it before the timer ran out" or "Nice try! Remember, you have to find it before the timer stops to win a sticker!"

Figure 2

Machines presented during Familiarization 1 and 2

Tube Familiarization: Children were introduced to two different y-shaped tubes: a fully transparent set and a fully opaque set (Figure 3). First, they saw a set of fully transparent tubes. The tubes were then attached to two empty, unoccluded machines and a yellow square was dropped from the top of the tubes into one of the machines in a pseudorandom order (LRRL). The experimenter pointed to the tubes and said, "Here, you will be able to see the square as it comes down." The child was instructed to point to the tube that the yellow square went down after each drop. Children were then shown a set of red, fully opaque tubes. The same procedure was repeated for these tubes, though the experimenter said, "Here, you will not be able to see the square as it comes down" when the tubes first appeared, and drops followed the pattern RLLR.

Figure 3

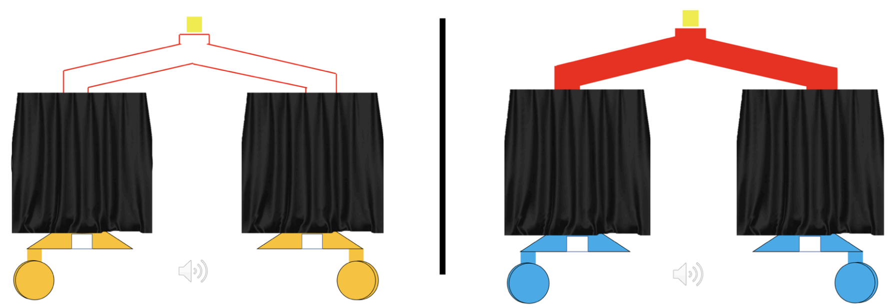


Tube Familiarization Configuration

Test Trial 1: Two new occluded machines appeared on the screen, this time with either the transparent or opaque tubes attached to the top (order counterbalanced across participants), and children were told to find the yellow square again (see Figure 4). This trial also included the 31 second timer, and children were told they can earn one of the 3D animal stickers if they succeed. Critically, there was not a yellow square in either of the machines during test trials; only blue marbles exited each machine. No prompts were given to children during test trials. After the trial ended, children were reminded that they must find the yellow square before the music stops in order to win a sticker.

Test Trial 2: Test Trial 2 followed the same procedure as Test Trial 1, but the other set of y-shaped tubes were attached to two new occluded machines (See Figure 4). At the end of this final test trial, children got to select one of the 3D animal stickers for their participation in the study.

Figure 4



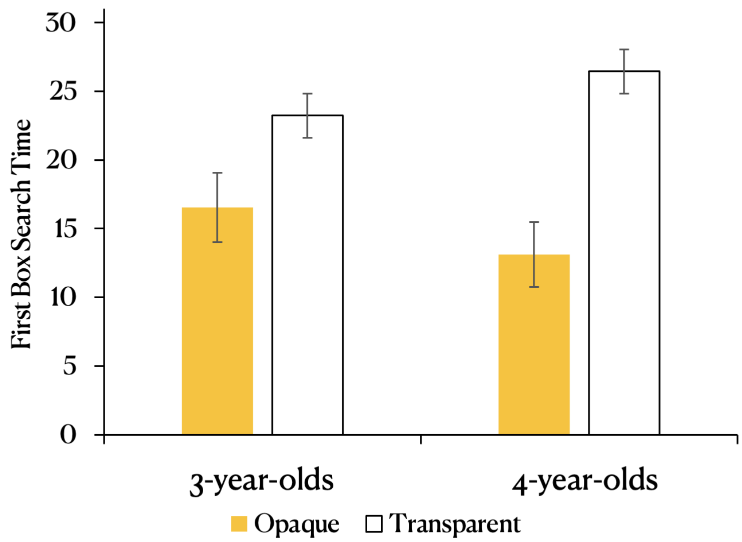
Machines presented during Test Trials

Data were coded with Datavyu (datavyu.org), a software that allows for frame-by-frame behaviorally coding. We recorded the amount of time children spent searching in the first machine, the total time children spent in each machine, the number of switches between machines, and which machine the child approached first during the transparent test trial. Children's first location searches were reliability coded, and coders reached a reliability score of 99.43%. Three 3-year-olds and one 4-year-old began searching in the incorrect search location in the transparent trial. These children were included in our analyses.

Experiment 2 Results

Statistical analyses were conducted using R (version 4.1.1). For both 3- and 4-year-olds, we examined whether there was a significant difference in children's search time between the first location they approached in the transparent trial (3-year-olds: M = 23.24, SD = 11.93; 4-year-olds: M = 26.46, SD = 9.59), compared to the opaque trial (3-year-olds: M = 16.53, SD = 12.37; 4-year-olds: M = 13.12, SD = 11.56), $t(23) = 3.2068$, $p = .003$ and $t(23) = 4.9259$, $p < .005$, respectively. (See Figure 5). Additionally, both 3- and 4-year-olds switched search locations significantly more in the opaque trial, compared to the transparent trial (3-year-olds: $t(23) = -2.1289$, $p = 0.04419$; 4-year-olds: $t(23) = -2.5293$, $p = .01874$, with no significant differences between the two age groups ($p = .973$). These results replicate the findings from Experiment 1 in a digital, iPad version of the task.

Figure 5



Three- and four-year-olds' search time in the first location they approached on transparent and opaque trials in Experiment 2. Error bars represent SEM.

Experiment 3

The findings from Experiment 1 and Experiment 2 provide promising support for the claim that preschoolers are sensitive to possibility concepts. In Experiment 3, we investigate an alternative hypothesis-- that children's successes on our previous tasks resulted from a difference in strength of representation between the two test trials. With the transparent tube, children were allowed direct visual access to the target object as it fell from the tube. In this condition, children's representation of necessity was reached via direct observation. Contrastingly, children did not have visual access to the object in the opaque trial. Here, children's representation of possibility was reached via inference. Previous work suggests that direct visual perception leads to a greater strength of representation (Call & Carpenter 2001). Therefore, it is plausible that children searched longer in the first box in the transparent trial due to this difference in representation. In Experiment 3, we address this potential limitation of the previous two experiments by matching the representation strength across the two test trials. We replace the fully transparent y-shaped tube with a half-transparent, half-opaque tube. Here, the target object always falls down the opaque side of the tube, not the transparent one, and children must infer its location as opposed to merely observing it. With this modification, children must use inference in order to guide their search behavior in both trials, thus representation strength is matched across conditions. We hypothesized if children's search behaviors in Experiment 1 and Experiment 2 are guided by representations of necessity and possibility respectively, children should perform identically to Experiment 1 and Experiment 2. In other words, performance on the half transparent, half opaque tube and the fully opaque set of tubes should be significantly different.

Experiment 3 Methods

Participants

The planned sample is 48 preschool aged children (data collection is ongoing with N= 12 3 year-olds and N= 12 4 year-olds). Children were recruited from the Early Learning and Cognition Lab's existing database and preschools in the greater San Diego area. All other participant criteria were the same as in Experiment 2.

Materials

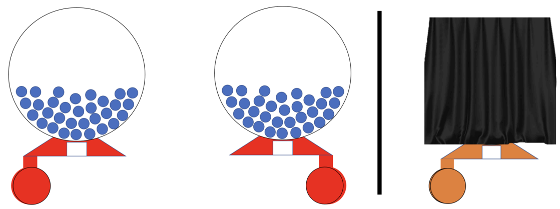
Materials for Experiment 3 were identical to Experiment 2.

Procedure

The procedure was mostly identical to Experiment 2, however included several modifications to better scaffold children's understanding.

Warm-up Phase: The Warm-up phase was identical to that of Experiment 2, however the transparent singular machine was removed from training such that children only saw two transparent machines followed by a singular occluded machine (See Figure 6).

Figure 6



Machines presented during warm-up phase

Familiarization 1: The procedure was identical to that of Experiment 2's Familiarization 1, however The yellow square was hidden in the right machine and was dispensed on the 4th press.

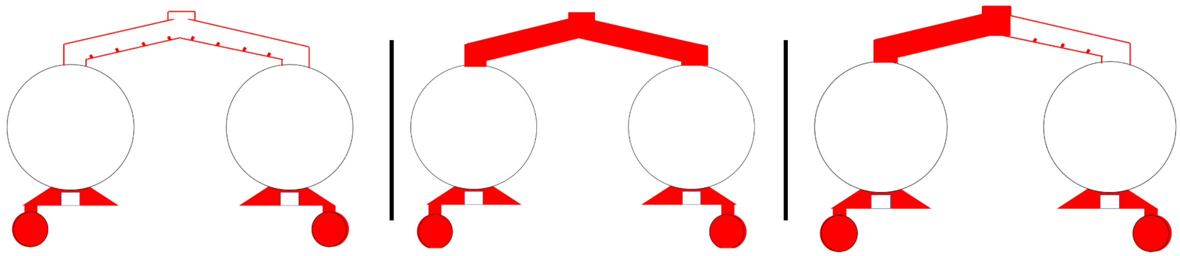
Familiarization 2: Familiarization 2 followed the same procedure as Familiarization 1, but the yellow square was instead hidden inside the left machine and was dispensed on the 5th press.

Familiarization 3: Familiarization 2 followed the same procedure as the first two familiarizations, but the yellow square was hidden inside of the left machine and was dispensed on the 5th press. This trial also introduced a 31 second musical timer. Protocol for feedback in this trial was the same as Experiment 2's *Familiarization 2*.

Familiarization 4: Familiarization 4 followed the same procedure as Familiarization 3, but the yellow square was instead hidden inside the right machine and was dispensed on the 6th press.

Tube Familiarization: This familiarization followed the same procedure as in Experiment 2, however children were introduced to three different y-shaped tubes: a fully transparent set, a fully opaque set, and a half-transparent, half-opaque set (Figure 7). With the fully transparent tubes, target object drops followed the pattern LRRL, the opaque tubes followed the pattern LRLR, and the half-transparent, half-opaque followed the pattern RLLR. The yellow square also made a musical noise as it fell down the tubes to better scaffold children's understanding.

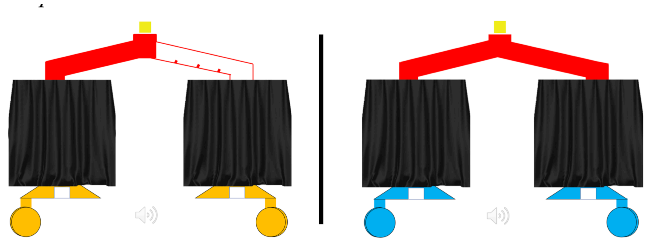
Figure 7



Order of Tube Familiarization

Test Trials: Protocol for test trials was identical to that of Experiment 2, however instead of seeing a fully transparent tube, children were presented with the half-transparent, half-opaque tube. Performance with the half-transparent tube was compared with children's performance with the fully opaque tube (See Figure 8).

Figure 8



Tubes presented during Test 1 and Test 2

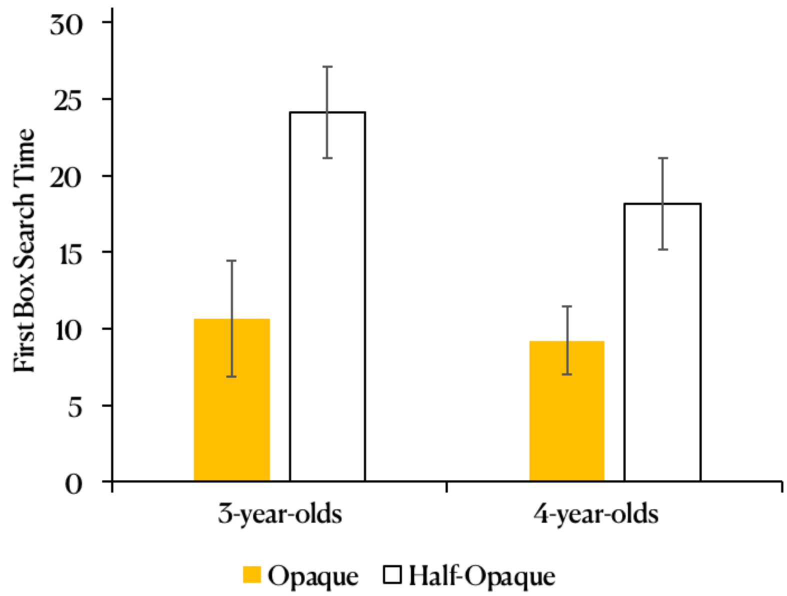
As in Experiment 2, data were coded with Datavyu (datavyu.org), a software that allows for frame-by-frame behaviorally coding. Children's first location searches were reliability coded, and coders reached a reliability score of 99.99%. Eight 3-year-olds and three 4-year-old began searching in the incorrect search location in the transparent trial (See General Discussion for potential explanations for this increased number). Children who began searching in the incorrect location were excluded from our preliminary analyses.

Experiment 3 Preliminary Results

Data was analyzed using R (version 4.1.1). For both 3- and 4-year-olds, we examined whether there was a significant difference in children's search time between the first location they approached in the half-opaque trial (3-year-olds: M = 24.10, SD = 11.94; 4-year-olds:

M = 18.14 , SD = 11.09), compared to the fully-opaque trial (3-year-olds: M = 10.63, SD = 10.01; 4-year-olds: M = 9.22, SD = 6.66), $t(6) = 2.55$, $p < .05$ and $t(8) = 2.41$, $p < .05$, respectively. (See Figure 9). These results suggest that the previous two experiments may reflect a true effect-- we see a significant difference in children's search behavior across the half-opaque and fully opaque trials.

Figure 9



Three- and four-year-olds' search time in the first location they approached on transparent and opaque trials in Experiment 3. Error bars represent SEM.

General Discussion

Across three separate experiments, we examine 3- and 4-year-olds' possibility reasoning through a novel search paradigm. In Experiment 1, we find that both 3- and 4-year-olds searched significantly longer in the first location they approached in the transparent trial, compared to the opaque one. Children also switched search locations significantly more in the opaque trial, relative to the transparent one, and there were no significant differences in search behavior between the two age groups. In Experiment 2, we replicate these same findings in a digital paradigm that does not require children's manual search and provides less sensory feedback. In Experiment 3, we investigate whether children's successful performance in Experiments 1 and 2 were a result of the low-level visual cues provided to children during the transparent tube test trial. By replacing the transparent tube with a half-opaque, half-transparent tube children had to infer the target object's location instead of directly observing it. Preliminary results replicate the findings from Experiments 1 and 2, however it is important to note that more 3- and 4-year-olds began searching in the incorrect search location in the half-opaque tube test trial, compared to the previous two experiments. Despite this difference, children's search patterns remained relatively the same as in the first two experiments, suggesting that younger children may struggle more with the principle of negation (Nordmeyer et al., 2016). This is something that we plan to address in future work. Taken together, these three experiments provide preliminary evidence against the claim that children are minimal representers of possibility, and that children as young as three are capable of discriminating between a necessary and merely possible outcome.

In both Experiments 1 and 2, we find main effects at the group level, however not at the individual level for both 3-and 4-year-olds. A select group of children exhibited the correct search pattern across test trials, suggesting that there is a variance in 3- and 4-year-olds' ability to understand modal concepts, and the capacity is early emerging within the preschool years. Future work may replicate these findings with larger sample sizes in order to better understand the developmental trajectory of acquiring possibility concepts at the individual level.

According to the minimal account, children simulate a single outcome and will treat that simulation as truth when presented with multiple possibilities (see Leahy & Carey 2020 for a full discussion). Children's failures on previous modal reasoning tasks have been taken as evidence for the claim that children are minimal representers of possibility. However, before fully accepting the minimal account, it is crucial to ensure that previous failures were due to children's lack of competence, not performance. Across our three experiments, we control for children's impulsiveness to explore. If children were truly unable to discriminate between necessary and merely possible outcomes, we should not see a difference between children's search behavior with an opaque tube versus a transparent tube (children will simulate necessary outcomes in both test trials). Instead, we find that children's search behavior between the transparent and opaque trials differ significantly in our tasks: children switched locations significantly faster in the opaque trial when there were two possible locations for the target object, compared to the transparent trial where there was only one necessary location. These findings suggest that children's previous failures on modal reasoning tasks may not be due to a lack of understanding possibility concepts, but rather due to the nature of the tasks themselves.

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